12PB25MAT03

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ERNAKULAM REGION - PRE BOARD EXAMINATION (2025-26)

CLASS XII MATHEMATICS SUBJECT CODE :041

DURATION: 3 Hours MAX MARKS :80

SECTION A (1 MARKS)

1. C. $\frac{x}{\sqrt{1+x^2}}$

2. C. $1-\alpha^2-\beta\gamma=0$

3. D. 0

4. B. AB $=-$BA

5. A. $\frac{2\pi}{3}$

6. D. $B^{-1}=\frac{1}{6}\,A$

7 . B.8

8 . C. $\frac{1}{x^3y}$.

9. A.$(-\infty,\ \ -\frac{1}{4}]$

10. B.3

11. C. 0

12. D. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

13. A. $\sqrt{7}$

14. B. $2\vec{a}^{\,2}$

15. D. $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

16. C. line segment EG

17. B. $2p=q$

18. A. $\frac{61}{63}$

19. C. (A) is true but (R) is false

20. C. (A) is true but (R) is false

SECTION B(2 MARKS)

21A. $y=\sin^{-1}(x^2-4)$

$\Rightarrow\ -1\le x^2-4\le 1$ ½

$\Rightarrow\ -1+4\le x^2\le 1+4$

$\Rightarrow\ 3 \leq x^2 \leq 5$ ½

$\Rightarrow \sqrt{3} \leq |x| \leq \sqrt{5}$ ½

$\Rightarrow x \epsilon \left[-\sqrt{5}, -\sqrt{3}\right] \cup [\sqrt{3}, \sqrt{5}]$ ½

OR

21B. $\sin^{-1}\{\sin\frac{13\pi}{7}\} = \sin^{-1}\{\sin(2\pi - \frac{\pi}{7})\}$ ½

$= \sin^{-1}\{\sin\frac{-\pi}{7}\}$, $\frac{-\pi}{7} \epsilon [-\frac{\pi}{2}, \frac{\pi}{2}]$, 1

$= -\frac{\pi}{7}$ ½

22. $e^y(x+1) = 1$,differentiating with respect to x

$\Rightarrow e^y + (x+1)e^y\frac{dy}{dx} = 0$ ½

.Dividing by e^y

$\Rightarrow 1 + (x+1)\ \frac{dy}{dx} = 0$ ½

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ ½

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} == (\frac{dy}{dx})^2$ ½

23. $f(x) = |x| - |x+1| = \begin{cases} 1, x < -1 \\ -2x - 1, -1 \leq x < 0 \\ -1, x \geq 0 \end{cases}$ ½

For checking continuity at $x = -1$ *and* $x = 0$ 1

Function is continuous for all values of x.Hence no discontinuous point exist. ½

24A. $I = \int cos^3 x e^{logsinx} dx = \int cos^3 x\ sinx\ dx$ ½

Put $cosx = t \Rightarrow -sinxdx = dt$

$\Rightarrow sinxdx = -dt$ ½

$I = -\int t^3\ dt = -\frac{t^4}{4} + C$ ½

$= \frac{-(cos^4 x)}{4} + C$ ½

OR

24B. For correct figure ½

Required area= $\int_0^{\frac{\pi}{2}} cosx\ dx + |\int_{\pi/2}^{\pi} cosx\ dx|$ ½

$= [\ sinx]_{0}^{\pi/2} + |[sinx]_{\frac{\pi}{2}}^{\pi}|$ ½

=2 sq units ½

25. $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$

$\vec{a} + \vec{b} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$ ½

$|\vec{a} + \vec{b}| = 13$ ½

Required unit vector = $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{1}{13}(4\hat{\imath} + 3\hat{\jmath} - 12\hat{k})$ 1

SECTION C (3 MARKS)

26A. $(cosx)^y = (cosy)^x$

Taking log both sides, $ylog(cosx) = xlog(cosy)$ ½

Differentiating w r t x on both sides,

$-y\ tanx + \log(cosx)\ \frac{dy}{dx} = -xtany\frac{dy}{dx} + \log\ (cosy)$ $1\frac{1}{2}$

$\frac{dy}{dx} = \frac{\log(cosy)+ytanx}{xtany+\log\ (cosx)}$ 1

OR

26B. $x = a(2\theta - sin2\theta)$

$\frac{dx}{d\theta} = a(2 - 2cos2\theta) = 2a(1 - cos2\theta) = 4asin^2\theta$ 1

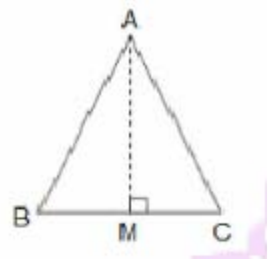
$y = a(1 - cos2\theta)$

$\frac{dy}{d\theta} = 2asin2\theta = 4asin\theta cos\theta$ 1

$\frac{dy}{dx} = cot\theta$ ½

at $\theta = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$ ½

27 Consider $BC = b$ be the fixed base and $AB = AC = x$ be the two sides of isosceles triangle.



½

$\frac{dx}{dt} = -3cm/s$

Area of triangle $A = \frac{1}{2}\ BC \times\ AM = \frac{1}{2} b\sqrt{x^2 - \frac{b^2}{4}}$ ½

$A = \frac{b}{4}\sqrt{4x^2 - b^2}$ ½

$\frac{dA}{dt} = \frac{b}{4}\ \frac{-8x}{2\sqrt{4x^2-b^2}}\frac{dx}{dt} = -\frac{3bx}{\sqrt{4x^2-b^2}}$ 1

When $x = b$, $\frac{dA}{dt} = -\sqrt{3}\,bcm^2/s$

Therefore area is decreasing at the rate of $\sqrt{3}\ bcm^2/s$ ½

28A.Given curve is $4x^2 + y^2 = 36$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{36} = 1$

Correct figure 1

Required area =4(area of region in first quadrant)

$= 4\int_0^3 2\sqrt{9 - x^2}dx$ ½

$= 8\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}(\frac{x}{3})\right]_0^3$

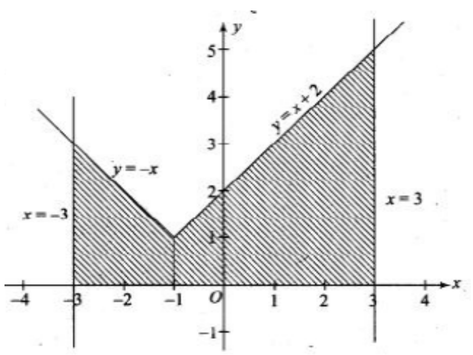
$=8 \times \frac{9}{2}\frac{\pi}{2}$ 1

$= 18\pi$ sq units ½

OR

28B. $y = |x + 1| + 1 = \begin{cases} x + 2\ , x \geq -1 \\ -x, x < -1 \end{cases}$

Correct figure 1



Required area $= \int_{-3}^{-1} -x\ dx + \int_{-1}^{3} x + 2\ dx$ ½

$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$ ½

$= \frac{1}{2}(1 - 9) + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right]$ ½

$= 16$sq units ½

29A .Any line through (2,1,3)can be written as $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \dots\dots..(1)$ ½

Where $a, b, c$are the direction ratios of line(1).

Line(1) is perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

DR'S of these lines (1,2,3) and $(-3,2,5)$respectively. ½

Then $a + b + c = 0 \;\; \dots.(2)\; and - 3a + 2b + 5c = 0 \dots..(3)$

Solving (2) and (3) $a = 2k\,, b = -7k\,, c = 4k$ 1 ½

Substituting in (1) ,required equation of the line is $\frac{x-2}{2k} = \frac{y-1}{-7k} = \frac{z-3}{4k}$

$\Rightarrow \;\; \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$ ½

OR

29B.Given lines are $\vec{r} = 4\hat{\imath} - \hat{\jmath} + s(\hat{\imath} + \widehat{2\jmath} - 3\hat{k})$ and $\vec{r} = \hat{\imath} - \hat{\jmath} + 2\hat{k} + t(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$

$\overrightarrow{a_1} = 4\hat{\imath} - \hat{\jmath}$, $\overrightarrow{b_1} = \hat{\imath} + \widehat{2\jmath} - 3\hat{k}$,$\overrightarrow{a_2} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\overrightarrow{b_2} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ ½

$\overrightarrow{a_2} - \overrightarrow{a_1} = -3\hat{\imath} + 2\hat{k}$ ½

$\overrightarrow{b_1} \times \overrightarrow{b_2} = 2\hat{\imath} - \hat{\jmath}$ ½

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{5}$ ½

$d = |\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}|$

$= 6/\sqrt{5}$ units 1

30.For sketching graph 1 ½

Corner points are $A(0,4)$ $B(5,0), C(6,0)$ $D(4,4)$ $E(0,6)$ ½

For finding maxmum value

Value of Z at $A = 160, B = 300$ $C = 360\,, D = 400$ $E = 240$ ½

Maximum value of $Z = 400$ at $D(4,4)$ ½

31. Let the events be:

Given $P(A) = \frac{4}{5}\,, P(B) = \frac{3}{4}\,, P(C) = \frac{2}{3}$

Then $P\bar{A} = \frac{1}{5}$ $, P(\bar{B}) = \frac{1}{4}$, $P(\bar{C}) = \frac{1}{3}$ 1

P(any two of them hit the target)= $P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$ 1

$= P(A).P(B)P(\bar{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C)$ ½

$= \frac{13}{30}$ ½

SECTION D(5 MARKS)

32. $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$|A| = 67$ ½

$\text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ 2

$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ ½

Given system of equation is $x + 2y - 3z = -4$; $2x + 3y + 2z = 2$ and $3x - 3y - 4z = 11$

In matrix form it can be written as $AX = B$,where

$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ½

$X = A^{-1}B = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ 1

$= \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

Thus $x = 3, y = -2, z = 1$ ½

33A. $I = \int \frac{x\tan^{-1}x}{(1+x^2)^{\frac{3}{2}}}\,dx = \int \frac{x\tan^{-1}x}{(1+x^2)\sqrt{(1+x^2}}\,dx$ ½

Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}\,dx = dt$ and $tant = x$ ½

$I = \int \frac{tant.t\,dt}{\sqrt{1+tan^2t}} = \int t\,sint\,dt$ 1

$= -tcost + \int cost\,dt$

$= sint - tcost + C$ 2

$= \frac{x}{\sqrt{1+x^2}} - \tan^{-1}x.(\frac{1}{\sqrt{1+x^2}}) + C$

$= \frac{1}{\sqrt{1+x^2}}[x - \tan^{-1}x] + C$ 1

33B. $I = \int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x}\,dx \ldots\ldots(1)$

$I = \int_0^{\pi} \frac{\pi - x}{a^2 cos^2(\pi - x) + b^2 sin^2(\pi - x)}\, dx$ ½

$= \int_0^{\pi} \frac{\pi - x}{a^2 cos^2 x + b^2 sin^2 x}\, dx \ldots\ldots (2)$ ½

Adding 2I $= \int_0^{\pi} \frac{\pi}{a^2 cos^2 x + b^2 sin^2 x}\, dx$ ½

$\mathrm{I} = \frac{\pi}{2}\, 2\int_0^{\frac{\pi}{2}} \frac{1}{a^2 cos^2 x + b^2 sin^2 x}\, dx$ ½

$I = \pi[\int_0^{\frac{\pi}{4}} \frac{1}{a^2 cos^2 x + b^2 sin^2 x}\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{a^2 cos^2 x + b^2 sin^2 x}\, dx]$ ½

$I = \pi[\int_0^{\frac{\pi}{4}} \frac{sec^2 x}{a^2 + b^2 tan^2 x}\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{cosec^2 x}{a^2 cot^2 x + b^2}\, dx]$ ½

Put $tanx = t$, $\cot x = u$

$I = \pi[\int_0^1 \frac{1}{a^2 + b^2 t^2}\, dt - \int_1^0 \frac{1}{a^2 u^2 + b^2}\, du]$ 1

$= \frac{\pi}{ab}[\tan^{-1}\frac{a}{b} + \tan^{-1}\frac{b}{a}] = \frac{\pi^2}{2ab}$ 1

34A. $x^2 dy + (xy + y^2)dx = 0$

$\frac{dy}{dx} = -\frac{xy + y^2}{x^2} \ldots\ldots (1)$ .it is a homogeneous differential equation ½

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (1)

$v + x\frac{dv}{dx} = -\frac{x\, vx + (vx)^2}{x^2}$

$= -(v + v^2)$

$x\frac{dv}{dx} = -2v - v^2$ 1 ½

$\int \frac{1}{v(v+2)} dv = -\int \frac{1}{x}\, dx$

$\Rightarrow \frac{1}{2}\int (\frac{1}{v} - \frac{1}{v+2})\, dv = -\int \frac{1}{x}\, dx$

$\Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -logx + \log C$ 1

$\Rightarrow \log\frac{v}{v+2} = 2\log\frac{C}{x}$

$\Rightarrow \frac{v}{v+2} = (\frac{c}{x})^2$

$\frac{y}{y+2x} = (\frac{c}{x})^2$ is the general solution 1

Given that $y = 1\ when\ x = 1$, $c^2 = \frac{1}{3}$

Particular solution is $\frac{y}{y+2x} = (\frac{1}{3x})^2$ 1

$\Rightarrow 3x^2 y = y + 2x$

34B .Given $(\tan^{-1} x - y)dx = (1 + x^2)dy$

$\frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{\tan^{-1}x}{1+x^2}$,is a linear differential equation ½

$P = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$ ½

IF= $e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x}$ ½

Solution is $y\, e^{\int P dx} = \int Q e^{\int P dx}\, dx + C$ ½

$\Rightarrow y e^{\tan^{-1} x} = \int \frac{\tan^{-1}x}{1+x^2} e^{\tan^{-1} x} dx$ ½

Put $t = \tan^{-1} x \Rightarrow dt = \frac{1}{1+x^2}\, dx$

$\Rightarrow y e^{\tan^{-1} x} = \int t\, e^t dt$

$= te^t - e^t + C$ 1 ½

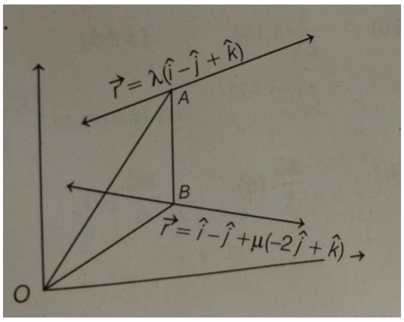
$\Rightarrow y e^{\tan^{-1} x} = \tan^{-1}x\; e^{\tan^{-1} x} - e^{\tan^{-1} x} + C$

$y e^{\tan^{-1} x}. = (\tan^{-1}x - 1)e^{\tan^{-1} x} + C$ 1

35.Given equation of lines are

$\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$.

These lines are not parallel as $\hat{\imath} - \hat{\jmath} + \hat{k}$ is not parallel to $-2\hat{\jmath} + \hat{k}$



½

Let AB be the shortest distance between the lines such that Ab is perpendicular to both the lines.

Let Position vector of the point A lying on the line $\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and Let the position vector of the point B lying on the line $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$.$=\hat{\imath} + (-1 - 2\mu)\hat{\jmath} + \mu\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (1 - \lambda)\hat{\imath} + (-1 - 2\mu + \lambda)\, \hat{\jmath} + (\mu - \lambda)\hat{k}$ 1

$\overrightarrow{AB}$ is perpendicular to both $\hat{\imath} - \hat{\jmath} + \hat{k}$ and $-2\hat{\jmath} + \hat{k}$

$\Rightarrow 2 + 3\mu - 3\lambda = 0 \ldots..(1)$ and $2 + 5\mu - 3\lambda = 0 \ldots\ldots(2)$

From (1) and (2) $\lambda = \frac{2}{3}$, $\mu = 0$ 2

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \frac{1}{3}\hat{\imath} - \frac{1}{3}\hat{\jmath} - \frac{2}{3}\hat{k}$ 1

$|\overrightarrow{AB}| = 2/\sqrt{3}$ units ½

SECTION E (4 MARKS)

36. I. $2^6 = 64$ 1

II. $G = \{g_1, g_2\}$

$R = \{(g_1, g_1), (g_2, g_2)\}$ 1

III A. $B = \{b_1, b_2, b_3\}$

$R_1 = \{(b_1, b_2), (b_2, b_3)\}$

Ordered pairs to be added to make R_1

(a) reflexive but not symmetric : $\{(b_1, b_1), (b_2, b_2), (b_3, b_3)\}$ 1

(b) reflexive and symmetric but not transitive.

$\{(b_1, b_1), (b_2, b_2), (b_3, b_3), ((b_2, b_1)(b_3, b_2)\}$ 1

III B. $x^2 = 4y$; where $0 \le x \le 20\sqrt{2}$ & $0 \le y \le 200$,

Let $y = \frac{x^2}{4}$ $in\ [0,20\sqrt{2}] \to [0,200]$

To prove one one

$f(x) = f(y) \Rightarrow \frac{x^2}{4} = \frac{y^2}{4}$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = \pm y$ ½

$x = -y$ is not possible as $0 \le x \le 20\sqrt{2}$

$\Rightarrow x = y$ ½

To prove on to

$y \in [0,200]$ and $y = \frac{x^2}{4}$

$\Rightarrow x = \pm 2\sqrt{y}$

Since $0 \le y \le 200$,for all $y \in [0,200]$ there exist a pre image in $x \in [0,20\sqrt{2}]$ 1

37.I Let x is the number of days after 1st July,

Price $= Rs\ (300 - 3x)$ ½

Quantity = 80 quintals+x (1 quintal per day) ½

II. $R(x) = quantity \times Price = (300 - 3x)(80 + x)$ ½

$= 24000 + 60x - 3x^2$ ½

III A. $R'(x) = 60 - 6x$ ½

$R'(x) = 0 \Rightarrow 60 - 6x = 0$

$x = 10$ 1

For maximum revenue , $R''(x) = -6 < 0$ ½

OR

III B. Pranav's father attains maximum revenue after 10days .So he should harvest the onions after 10days of 1st July ,ie on 11 th July. 1

Maximum revenue = $R(10) = Rs\ 24300$ 1

38.Given $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$

$P\left(\frac{L}{A}\right) = \frac{24}{100}$, $P\left(\frac{L}{B}\right) = \frac{22}{100}$, $P\left(\frac{L}{C}\right) = \frac{17}{100}$, $P\left(\frac{L}{D}\right) = \frac{9}{100}$

I. $P\left(\frac{A}{L}\right) = \frac{P(A)P\left(\frac{L}{A}\right)}{P(A)P\left(\frac{L}{A}\right)+P(B)P\left(\frac{L}{B}\right)+P(C)P\left(\frac{L}{C}\right)+P(D)P\left(\frac{L}{D}\right)}$ ½

$= \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}}$ 1

$= \frac{1}{3}$ ½

II. Let S = Event that exactly one of the parent is left handed

$P\left(\frac{L}{S}\right) = \frac{P(L\cap S)}{P(S)}$ 1

$= \frac{\frac{12}{100}}{\frac{39}{100}} = \frac{12}{39}$ 1